Hypercomplex Interest

An absolutely scuffed bastardization of math and economics

ArgentumCation (Mira)

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An absolutely scuffed bastardization of math and economics

You know what, fuck you

rotates your interest rates 90°

You know what, fuck you

Compound Interest

$z = Pe^{rt}$

- P: Principal
- r: Interest rate
- t: Time
- z: Money

1. So I'm sure y'all remember this formula from like 4th grade
2. I barely do so here's a refresh
3. P is your principal, or how much money you initially put in or took out
4. r is the interest rate, you want this to be low if you're borrowing and high if you're lending
5. t is time, unless you have a TARDIS, this one is pretty out of your control
6. z is how much money you owe/are owed, I know this isn't the standard variable name but bear with me

Compound Interest

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Now let's make it spicy

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Complex Interest

- Starting value is \$2
- We rotate it 45° in $\mathbb{C}$
- ???
- Wait that's not profit

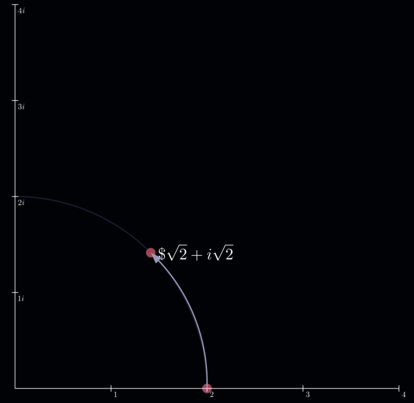
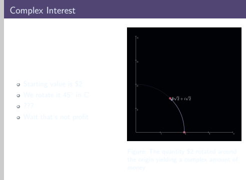


Figure: The quantity \$2 rotated around the origin yielding a complex amount of money



1. Who says you need to have a real number for your interest rate?
2. The Fed moves rates up and down all the time, and I think it's high time those cowards start moving it left and right
3. But first, what does it mean to move money to the left? [Vsauce Sting]
4. For now let's try rotating it around a circle,
5. So we take \$2 and rotate it around the origin 45 degrees in the complex plane
6. We get $\sqrt{2} + i\sqrt{2}$

- Complex numbers can also be represented as $z = Pe^{i\theta}$

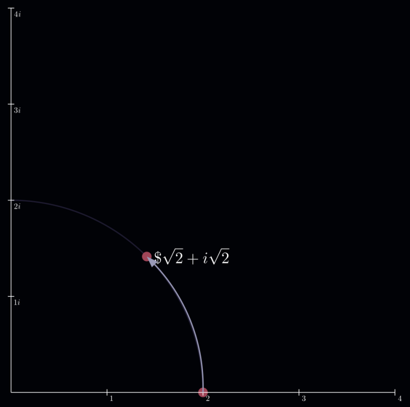
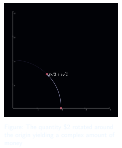


Figure: The quantity $2 rotated around the origin yielding a complex amount of money



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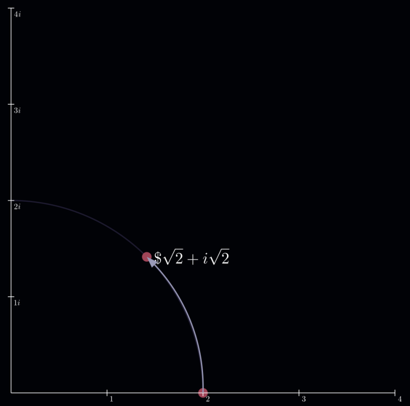
- $z = Pe^{i\theta t}$



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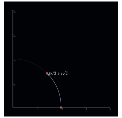

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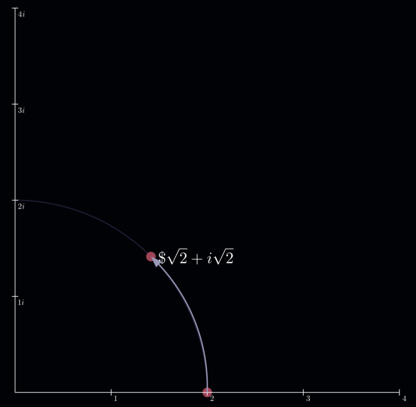

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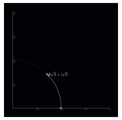

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4. and now we can see that $i\theta$ acts like an imaginary interest rate
5. but like what the fuck does this mean, does MOHELA owe me imaginary money or something?
6. well I left out an important detail

- Complex numbers can also be represented as $z = Pe^{i\theta}$
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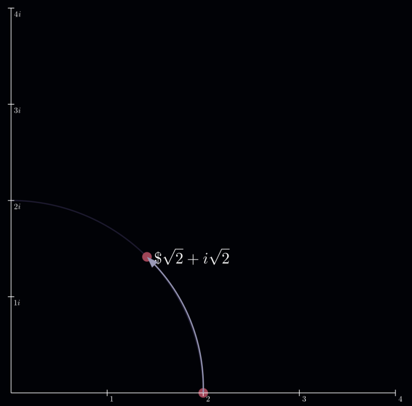

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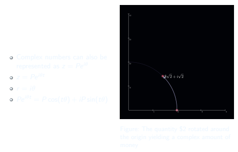

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Euler's Identity

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- But why?

Euler's Identity

1. Do the proof on the whiteboard

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- But why?
- It's Taylor Series time

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- Well the real part ends up being

$$\Re(z) = P\cos(\frac{\pi}{2}rt)$$

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3. So if I wait about 30 years after my initial loan, I can owe none money with left bread, if I wait 60 years, MOHELA needs to double my money for free.
4. Honestly I think the stock market is a better investment strategy. But that seems a bit too based either way, so no wonder I can't get my loans refinanced like this

Split-Complex Interest Rates

Morpheus Voice: What if I told you there's more than one way to rotate a cow?

Note

Listen closely cause this next part got me laid

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Split-Complex Interest Rates

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- But what if you did the opposite?

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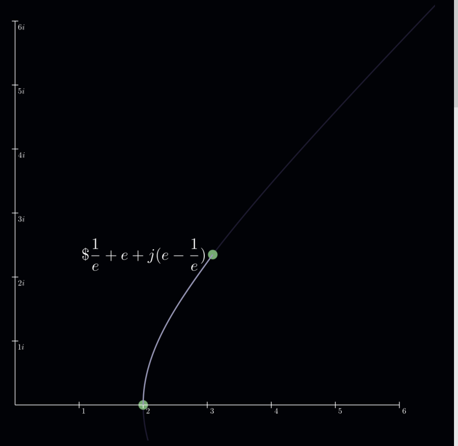
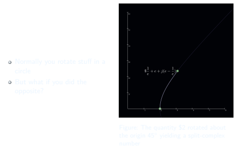


Figure: The quantity $2 rotated about the origin 45° yielding a split-complex number



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Introducing: The Hyperbola

- The unit circle is defined by $x^2 + y^2 = 1$
- The unit hyperbola is defined by $x^2 - y^2 = 1$
- sin and cos have sinh and cosh as hyperbolic equivalents
- i has j as it's hyperbolic equivalent

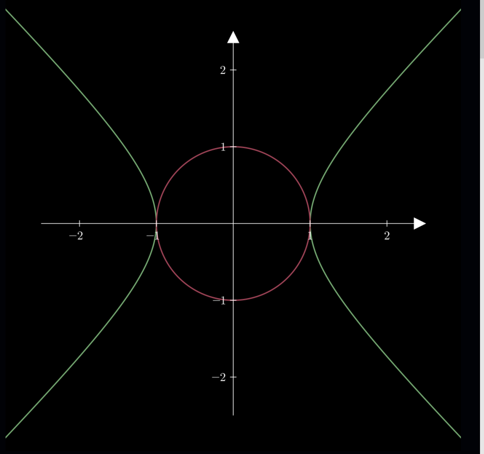
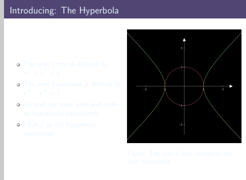


Figure: The Unit Circle, alongside the Unit Hyperbola



1. So with the unit circle we had an equation like $x^2 + y^2 = 1$, well there's a unit Hyperbola too and the equation for that is $x^2 - y^2 = 1$
2. There's actually a lot of circle stuff that have hyperbolic equivalents
3. Remember how sin and cos can turn an angle into x and y coordinates on a circle? There's also sinh and cosh which can turn an angle into coordinates on a hyperbola
4. Here's the big one we care about though, multiplying something by i rotates it around a circle, there's actually a constant j that rotates something around a hyperbola

The Split-Complex constant: j

- What exactly is j thought?

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└─Split-Complex Interest
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- $j = \sqrt{1}$

The Split-Complex constant: j

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2. (Wait for someone to call you out on your bullshit)

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- Okay so then what does $e^{\theta j}$ break down into?

Well I wouldn't be posing this question if there wasn't a semi-interesting answer

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5. Do the proof on the board

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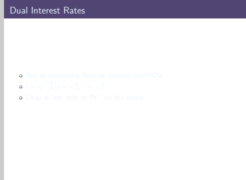
6. As a student loan payer, this is worse than a real-valued interest rate

Dual Interest Rates

- Not as interesting from an interest rate POV
- $i = \sqrt{-1}, j = \sqrt{1}, \varepsilon = \sqrt{0}$
- Okay so lets look at $Pe^{\varepsilon i}$ on the board



1. Okay this one isn't as interesting from an interest rate point-of-view, but I need to include it for completeness
2. I guess k was taken when they named this one?

Automatic Differentiation

- Let's try replacing x with $x + \varepsilon$ in some functions
- $f(x) = x^2$

- $f(x) = \sin(x)$

1. While we're here lets try plugging $x + e$ into functions for shits and giggles

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- $f(x) = x^2 = 2x\varepsilon$
- $f(x) = \sin(x) = \sin(x) + \varepsilon \cos(x)$
- $f(x + \varepsilon) = f(x) + \varepsilon f'(x)$

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- But why?
- Remember from 9th grade

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

1. ε acts as an infinitesimal, so we can actually replace dx with it

- But why?
- Remember from 9th grade

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- $f'(x) = \frac{f(x+dx)-f(x)}{dx}$

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- $f'(x) = \frac{f(x+dx)-f(x)}{dx}$
- $f'(x) = \frac{f(x+\varepsilon)-f(x)}{\varepsilon}$
- $\varepsilon f'(x) = f(x+\varepsilon) - f(x)$

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Matrix Representations

- $i = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \to i^2 = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
- $j = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \to j^2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- $\varepsilon = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \to \varepsilon^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

Matrix Representations

1. Fun fact you can also represent the hypercomplex numbers as matrices
2. Note how all 3 of these have zeroes in the major diagonal, so we can also add them to real numbers by multiplying them by the identity matrix

Honey, I broke the concept of division!

- I mentioned earlier that you can't divide by ε, well you can't actually divide by j either
- Multiplying two nonzero items in the Split-Complex or Dual numbers has the potential to result in division by zero

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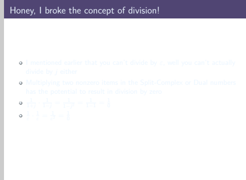
- Multiplying two nonzero items in the Split-Complex or Dual numbers has the potential to result in division by zero
- $\frac{1}{1+j} \cdot \frac{1}{1-j} = \frac{1}{1-j^2} = \frac{1}{1-1} = \frac{1}{0}$
- $\frac{1}{\varepsilon} \cdot \frac{1}{\varepsilon} = \frac{1}{\varepsilon^2} = \frac{1}{0}$



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